Technical Note 1970-42

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A New Technique for the Design of Non-Recursive Digital Filters

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LINCOLN LABORATORY

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Group 64

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ABSTRACT

An algorithm for designing non-recursive frequency-selective digital filters having equal stop-band and pass-band ripples of specified magnitude is described. There is no known upper limit on the order of the filter that can be designed by this algorithm and filters with orders in the hundreds already have been designed.

Accepted for the Air Force
Joseph R. Waterman, Lt. Col., USAF
Chief, Lincoln Laboratory Project Office

A NEW TECHNIQUE FOR THE
DESIGN OF NON-RECURSIVE DIGITAL FILTERS

The purpose of this note is to give a preliminary description of some new results in the design of non-recursive, digital filters. The specific filter types under consideration are low-pass, band-pass, band-reject, etc., although there are reasons to believe that the technique to be described can be extended to more general types of filters.

An excellent survey of the state-of-the-art of non-recursive filter design is given in a recent paper by Rabiner[1]. Another useful reference in this connection is Rabiner, Gold and McGonegal[2]. The present discussion is concerned with a much improved method of designing linear-phase, equi-ripple filters of the type proposed by Herrmann and Schuessler[3,4]. These filters are designed to have a specified number of ripples in the pass-band (N_p) and in the stop-band (N_s). In addition, the magnitude of the ripple is specified in the pass-band ($1 \pm \delta_1$) and in the stop-band ($\pm \delta_2$). The equi-ripple filter meeting these requirements is one whose frequency response has $N_p + N_s$ extrema that achieve the maximum allowable ripple values. An example is shown in Fig. 1. In this figure, $H(e^{j2\pi fT})$ is the frequency response of the filter at frequency f and T is the time-domain sampling interval.

Since the frequency response of a non-recursive digital filter is a trigonometric polynomial, Herrmann and Schuessler attempted to solve the equi-ripple filter design problem by writing a set of $2(N_p + N_s - 1)$ non-linear equations that force an $(N_p + N_s - 1)^{th}$ order polynomial to have extrema that achieve the maximum allowable ripple values. These equations are then solved by means of a non-linear programming technique which, unfortunately, fails to converge when the order of the polynomial is larger than about twenty. This is the main drawback to their method. The design algorithm that now will be described has no known limitation

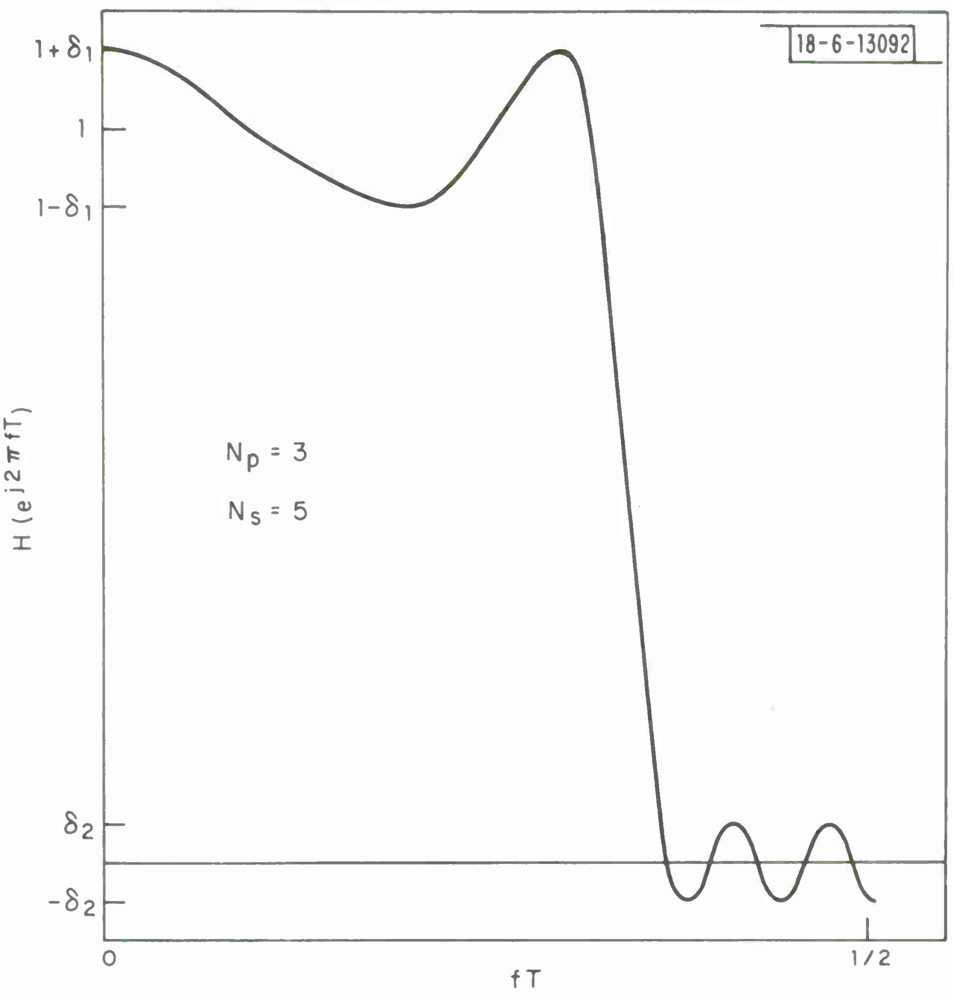

Fig. 1. An example of an equi-ripple filter.

on order and filters having orders in the hundreds already have been designed with it.

The proposed design algorithm is basically an iterative technique for producing a polynomial that has extrema of preassigned value. It begins by making an initial estimate of the frequencies at which these extrema will occur and then uses the Lagrange interpolation formula[5] to obtain a polynomial that goes through the maximum allowable ripple values at these frequencies. The details of this initial guess are not important; the author has been using a uniformly spaced set of frequencies ranging from 0 to 0.5 and requiring the polynomial to alternatingly go through $1 \pm \delta_1$ in the pass-band and $\pm \delta_2$ in the stop-band. This initial set of frequencies along with the associated Lagrange interpolation polynomial are sketched in Fig. 2. Note that the polynomial associated with the initial guess does not have extrema that achieve the maximum allowable ripples but, rather, extrema that exceed these values. The next stage of the algorithm is to locate the frequencies at which the extrema of the first Lagrange interpolation polynomial occur. These frequencies are taken to be a second, hopefully improved, guess as to the frequencies at which the extrema of the filter response will achieve the desired ripple values. This second set of frequencies is indicated in Fig. 2. The algorithm now "closes the loop" by using these new frequencies to construct a Lagrange interpolation polynomial that achieves the desired ripple values at these frequencies. The extrema of this new polynomial are then located and used to start the next cycle of the algorithm. The algorithm is reminiscent of, but different from, the Remes exchange algorithm[6] used in the theory of Tchebycheff approximation.

The formal, mathematical description of the design algorithm can be written as follows. Denote the frequencies obtained at the i^{th} stage of the algorithm by $f_k^{(i)}$, $k = 1, 2 \ldots N_p + N_s$, $f_1^{(i)} = 0$, $f_{N_p + N_s}^{(i)} = 1/2T$. The frequency response of

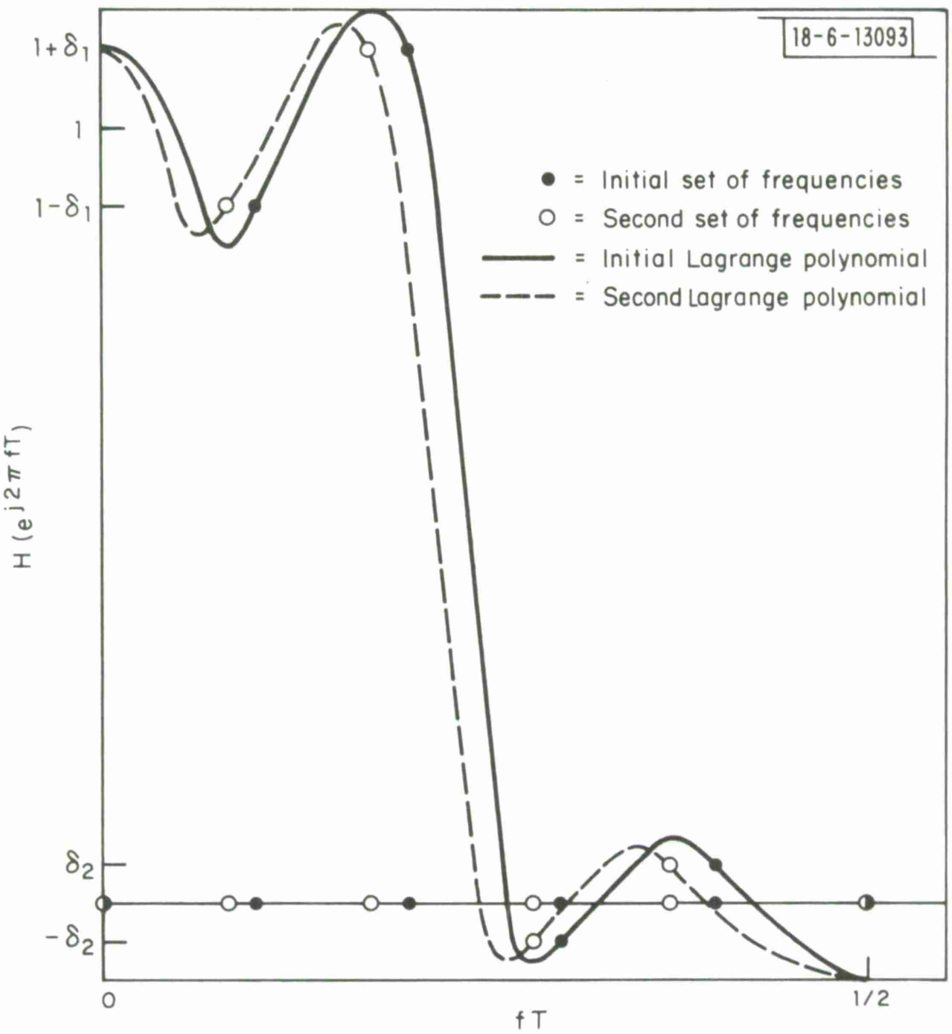

Fig. 2. The design algorithm for $N_p = N_s = 3$.

the filter at the i^{th} stage of the algorithm is then given by

$$G^{(i)}(f) = \frac{\displaystyle\sum_{k=1}^{N_p+N_s} \frac{A_k^{(i)}\ Y_k}{(X - X_k^{(i)})}}{\displaystyle\sum_{k=1}^{N_p+N_s} \frac{A_k^{(i)}}{(X - X_k^{(i)})}}$$

where

$$X = \cos 2\pi fT,\ X_k^{(i)} = \cos 2\pi f_k^{(i)} T,\ G^{(i)}(f) = H^{(i)}(e^{j2\pi fT})$$

$$A_k^{(i)} = \left[\prod_{\substack{e=1 \\ e \neq k}}^{N_p+N_s} (X_k^{(i)} - X_e^{(i)})\right]^{-1}$$

and $Y_k = 1 \pm \delta_1$ for subscripts referring to the pass-band and $Y_k = \pm \delta_2$ for subscripts referring to the stop-band. The frequencies associated with the $(i + 1)^{st}$ stage of the algorithm are now defined to be the $N_p + N_s$ solutions to the equations

$$\frac{d\,G^{(i)}(f_k^{(i+1)})}{df} = 0\ ,\ k=1,\ldots N_p + N_s$$

defining the $N_p + N_s$ extrema of $G^{(i)}(f)$. The $(i+1)^{st}$ frequency response $G^{(i+1)}(f)$ can now be obtained by replacing $X_k^{(i)}$ by X_k^{i+1} in the above equation. This process

is then repeated until the desired degree of convergence has been obtained.

It should be obvious that, if the procedure just described converges it must yield the desired equi-ripple filter. The algorithm has been programmed by the author on a Hewlett-Packard 9100A desk calculator and on the I.B.M. 360 computer by J. Siegal, an M.I.T. graduate student. On both machines the algorithm has been demonstrated to converge to within a preset accuracy limit in a surprisingly small number of iterations. An example of a 41^{st} order band-pass filter designed by the Hewlett-Packard program is shown in Fig. 3 and an example of a 251^{st} order low-pass filter designed by the I.B.M. 360 program is shown in Fig. 4. The latter design required 12 iterations.

There are a number of interesting mathematical conjectures that can be made about equi-ripple filters and the above described algorithm for designing them. The author in conjunction with Prof. A. Oppenheim of MIT believe they can prove these conjectures; however, full details are not available at this time. They will be published as a paper elsewhere in the literature in the near future. Because of the pertinence of these conjectures to the above algorithm, it seems appropriate to state the more important of them here. They are as follows:

1. The equi-ripple filter is optimum in the sense that no other non-recursive filter having the same number of pass-band and stop-band ripples and whose ripples are no greater than $1 \pm \delta_1$ and $\pm \delta_2$ in the pass-band and stop-band respectively can have a smaller transition-band. (The transition-band is defined as that interval of frequencies for which $\delta_2 < H(f) < 1 - \delta_1$.)

2. The algorithm described above always converges to that unique equi-ripple filter meeting the pass-band and stop-band specifications.

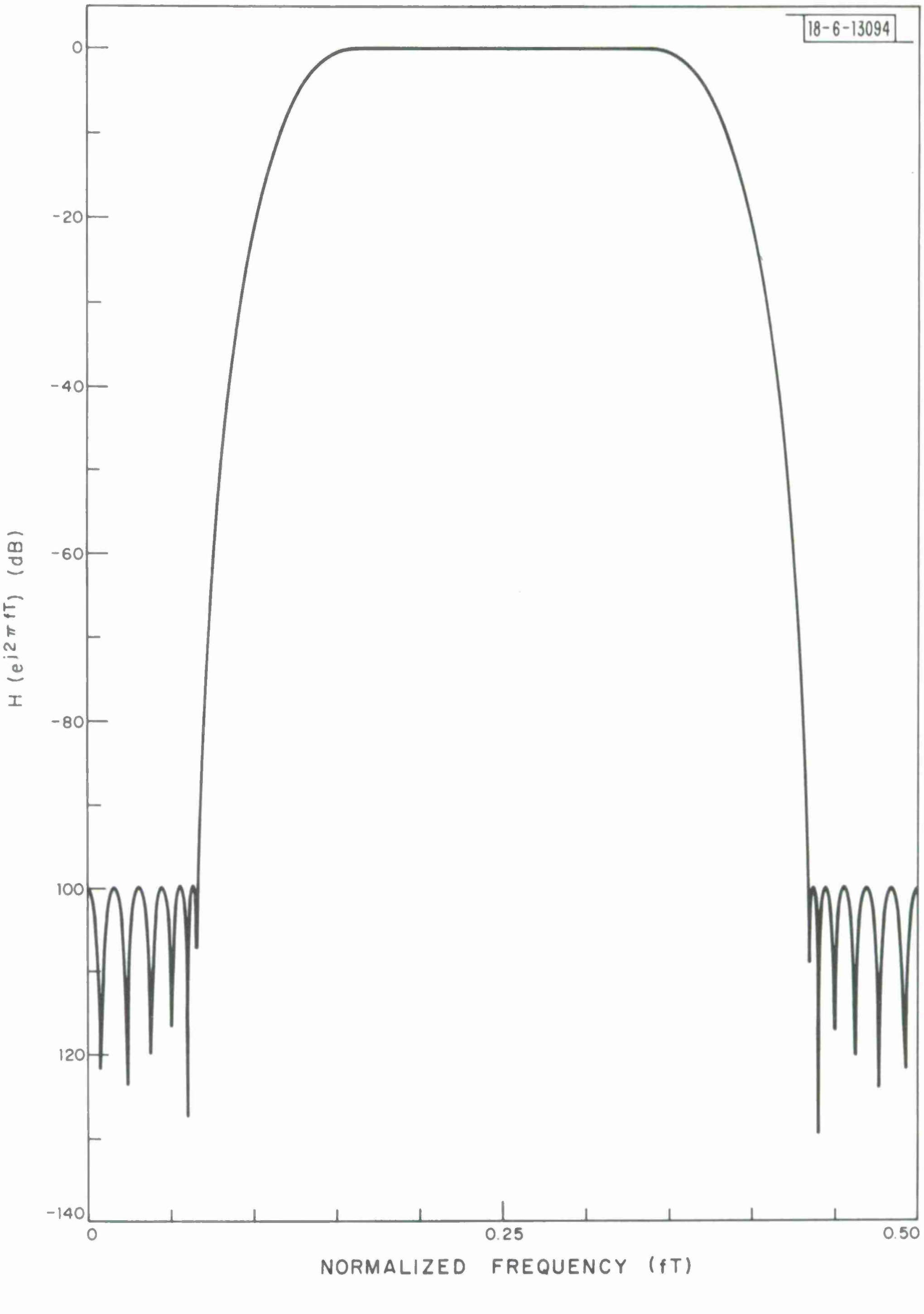

Fig. 3. A 41st order band-pass filter designed for $N_p = 9$, $N_s = 12$, $\delta_1 = .005$, $\delta_2 = 10^{-5}$.

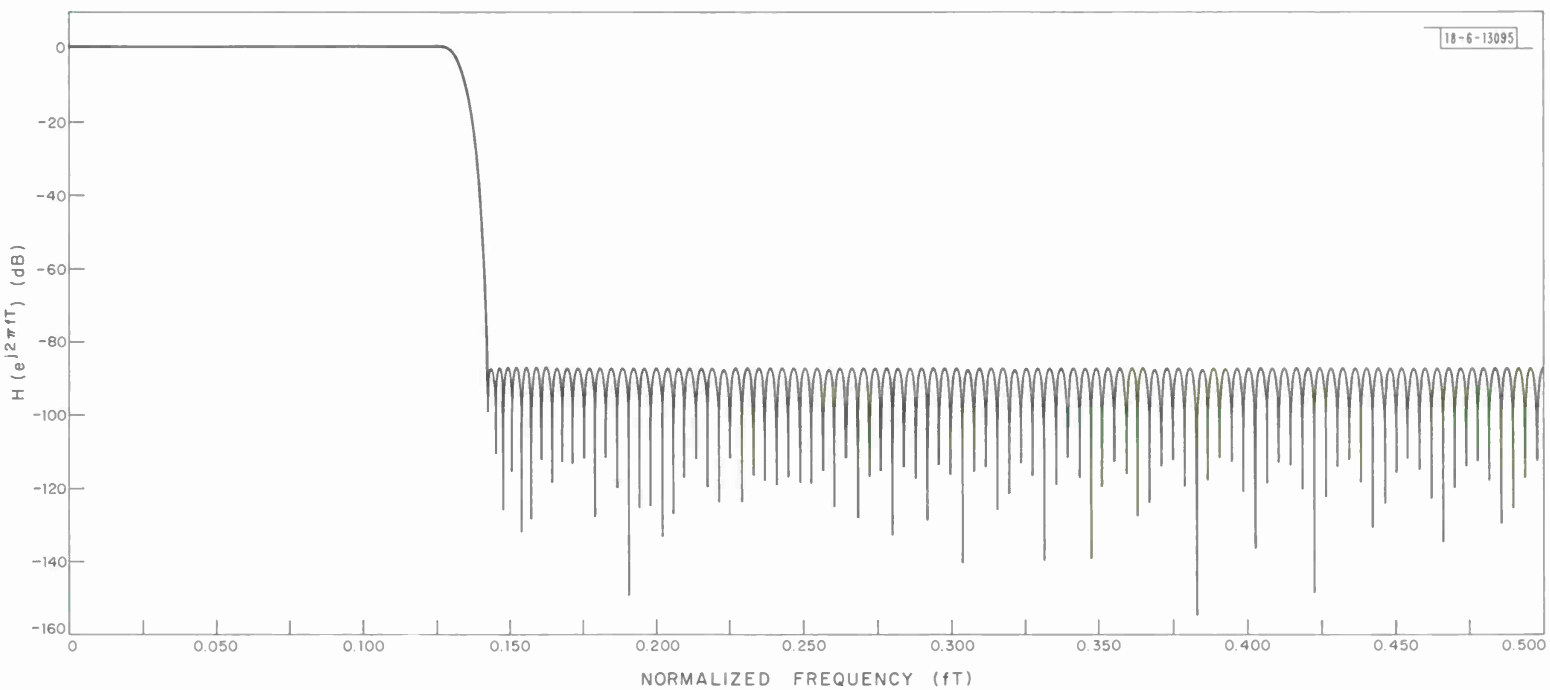

Fig. 4. A 251st order low-pass filter designed for $N_p = 32$, $N_s = 94$, $\delta_1 = .01$, $\delta_2 = 4 \times 10^{-5}$.

REFERENCES

1. L. Rabiner, Considerations in Finite Duration Impulse Response Digital Filter Design, NEREM record, 1970, pp. 148-149.

2. L. Rabiner, B. Gold and C. A. McGonegal, An Approach to the Approximation Problem for Non-recursive Digital Filters.

3. O. Herrmann and H. W. Schuessler, On the Design of Selective Non-recursive Digital Filters, IEEE Arden House Workshop, 1970, Harriman, N.Y.

4. O. Herrmann, On the Design of Non-recursive Digital Filters with Linear Phase, Electronics Letters, 1970.

5. R. W. Hamming, Numerical Methods for Scientists and Engineers McGraw Hill Book Co., Inc., New York, 1962, pp. 94-97.

6. J. R. Rice, The Approximation of Functions, Addison-Wesley Publishing Co., Reading, Mass. 1964, pp. 176-177.

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An algorithm for designing non-recursive frequency-selective digital filters having equal stop-band and pass-band ripples of specified magnitude is described. There is no known upper limit on the order of the filter that can be designed by this algorithm and filters with orders in the hundreds already have been designed.

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